Chapter 6 Time-Varying Fields

$$\nabla \cdot D = \rho_v \longrightarrow \oint_S \vec{D} \cdot d\vec{s} = Q$$

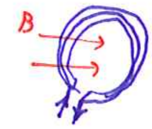
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \longrightarrow \oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \longrightarrow \oint_C \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s}$$

Electromotive force in a closed loop:

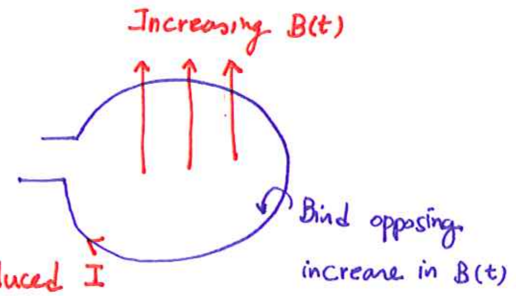
$$V_{emf} = -N \frac{d\phi}{dt} = -N \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$ Faraday's law



Case 1: Stationary Loop in a Time-varying Magnetic Field:

$$V_{emf}^{tr} = -N \int_S \frac{\partial \vec{B}}{\partial t} \cdot ds$$

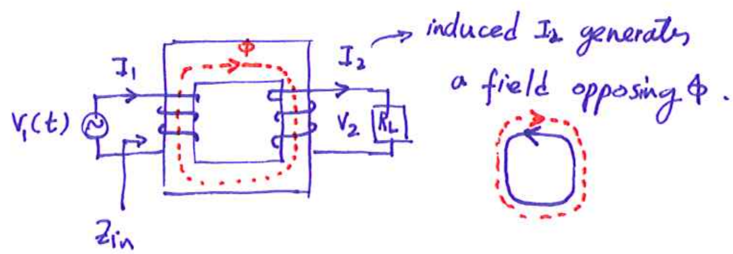
The polarity is governed by Lenz's law



Transformer:

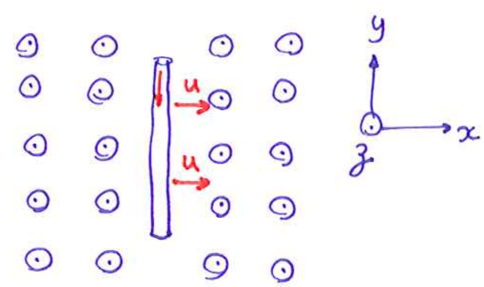
$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$



Case 2: Moving Conductor in a static Magnetic Field:

$$V_{emf}^{m} = \oint_C (\vec{U} \times \vec{B}) \cdot d\vec{l}$$

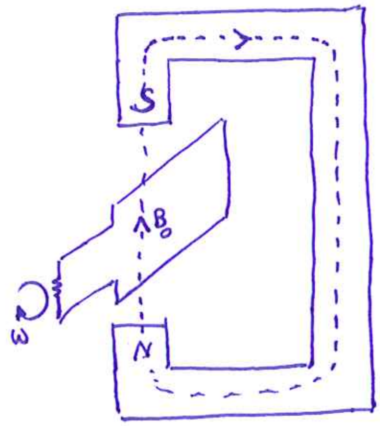


EM Generator:

$$V_{emf}^{m} = A\omega B_0 \sin(\omega t + C_0)$$

Area of the loop

Some constant for the reference at start $\alpha = \omega t + C_0$



Case 3: Moving Conductor in a Time-varying Field:

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

Displacement Current: $J_d \triangleq \frac{\partial \vec{D}}{\partial t} \rightarrow I_d = \int \vec{J_d}\cdot\vec{ds} = \int \frac{\partial \vec{D}}{\partial t}\cdot\vec{ds}$

Amper's law: $\oint_C \vec{H}\cdot\vec{dl} = I = I_C + I_d$

Charge current Continuity: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t} \rightarrow \int \vec{J}\cdot\vec{ds} = 0$ if $\frac{\partial \rho_v}{\partial t} = 0$

Electromagnetic Potentials: $\vec{B} = \vec{\nabla}\times\vec{A}$ valid for dynamic & static fields

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

Time-Harmonic Potentials: $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \vec{\nabla}\times\tilde{\vec{H}} = \tilde{\vec{J}} + j\omega\varepsilon\tilde{\vec{E}}$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla}\times\tilde{\vec{E}} = -j\omega\mu\tilde{\vec{H}}$$

Chapter 7 Unbounded EM waves

Time Harmonic fields:

$$\vec{\nabla}\cdot\vec{D} = \rho_v \rightarrow \vec{\nabla}\cdot\vec{\tilde{E}} = \frac{\tilde{\rho}_v}{\varepsilon}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \vec{\nabla}\times\vec{\tilde{E}} = -j\omega\mu\vec{\tilde{H}}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \rightarrow \vec{\nabla}\cdot\vec{\tilde{H}} = 0$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow \vec{\nabla}\times\vec{\tilde{H}} = \vec{\tilde{J}} + j\omega\varepsilon\vec{E}$$

Complex permitivity:

$$\vec{\nabla}\times\vec{\tilde{H}} = \vec{\tilde{J}} + j\omega\varepsilon\vec{\tilde{E}} = \sigma\vec{\tilde{E}} + j\omega\varepsilon\vec{E} = j\omega\overbrace{\left(\varepsilon - j\frac{\sigma}{\omega}\right)}^{\varepsilon_c}\vec{\tilde{E}}$$

$$\varepsilon_c \triangleq \underbrace{\varepsilon}_{\varepsilon'} - j\underbrace{\frac{\sigma}{\omega}}_{\varepsilon'' = \frac{\sigma}{\omega}} = \varepsilon' - j\varepsilon''$$

$$\Rightarrow \vec{\nabla}\times\vec{\tilde{H}} = j\omega\varepsilon_c\vec{\tilde{E}}$$

Waves in charge-free Medium:

$$\left.\begin{aligned} &\vec{\nabla}\cdot\vec{\tilde{E}} = 0 \\ &\vec{\nabla}\times\vec{\tilde{E}} = -j\omega\mu\vec{\tilde{H}} \\ &\vec{\nabla}\cdot\vec{\tilde{H}} = 0 \\ &\vec{\nabla}\times\vec{\tilde{H}} = j\omega\varepsilon_c\vec{\tilde{E}} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &\nabla^2\vec{\tilde{E}} - \gamma^2\vec{\tilde{E}} = 0 \\ &\nabla^2\vec{\tilde{H}} - \gamma^2\vec{\tilde{H}} = 0 \end{aligned}\right.$$

$\gamma^2 \triangleq -\omega^2\mu\varepsilon_c$ propagation constant